Numerical analysis of shells and space structures.

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1.-INTRODUCTION

Since the advent of the computer into the engineering field, the application of the numerical methods to the solution of engineering problems has grown very rapidly. Among the different computer methods of structural analysis the Finite Element (FEM) has been predominantly used.

Shells and space structures are very attractive and have been constructed to solve a large variety of functional problems (roofs, industrial building, aqueducts, reservoirs, footings etc). In this type of structures aesthetics, structural efficiency and concept play a very important role. This class of structures can be divided into three main groups, namely continuous (concrete) shells, space frames and tension (fabric, pneumatic, cable etc) structures.

In the following only the current applications of the FEM to the analysis of continuous shell structures will be discussed. However, some of the comments on this class of shells can be also applied to some extend to the others, but obviously specific computational problems will be restricted to the continuous shells.

Different aspects, such as, the type of elements, input-output computational techniques etc, of the analysis of shells by the FEM will be described below. Clearly, the improvements and developments occurring in general for the FEM since its first appearance in the fifties have had a significative impact on the particular class of structures under discussion.

2.-ELEMENTS

In the general linear shell analysis there are three main groups of elements: (a) flat elements, that combine the plane stress elements with the plate bending elements, i.e, elements C-0 and C-1. (b) curved shell elements created according to a general shell theory. (c) degenerated elements from 2-D and 3-D elastic elements constraining some specific shell degrees of freedom.

Special comments should be made regarding those elements which can not be included in any of the above groups.They have created in order to avoid the difficulties appearing in the stiffness assembly of plane elements due the different expansions used for the C-0 displacements and C-1 displacements. With the introduction of mid side-nodes these elements have been derived by Irons[1] with the name of semiloof shell elements, and they can perform quite well in several instances.

A recent review of all these elements has been published by Schnobrich in reference[2] Here, some comments about their suitability related to the type of analysis will be given.

With respect to the flat shell elements, the fulfilment of the Kirchhoff assumption implies the use of high order interpolation polynomial basis[3] There exist several techniques to create compatible C-1 elements[4] but they demand a large computational effort[5]. Moreover, in plasticity analysis this type of elements produce peculiar effects over a large part of the shell, particularly in non-smooth shells if excessive continuity (hyperelements) are considered. For that reason their use is mainly restricted to linear problems.

In order to construct curved shell elements, a consistent shell theory should be used:Shallow shell, curved plate, Vlasov, Goldenweizer etc. In general, an unique linear formulation of shells does not exist. In the development of the displacement field for a curved element some extra difficulties arise, when the rigid body movement, constant extensional and bending strains criteria have to be satisfied in order to obtain convergence. For particular shell geometries it is possible to satisfy these convergence requirements[6].Current interest on the research of these type of elements has diminished due to the inherent problems of convergence, interlocking and difficulties to model the boundary geometry.

The use of degenerated 2-D elements in shell analysis (C-0 elements) in the framework of the Mindlin/Reissner theories often leads to improved numerical behaviour in problems with material nonlinearities in comparison to the C-1 type of elements. Typical elements of this class are based on displacement finite element formulation. Examples of quadrilateral elements are variants of the 9-node biquadratic Lagrange element, e.g. using selective/reduced integration[7], the γ-stabilization method[8] , assumption of strain methods in the form of natural strains[9] or tensorial covariants strains[10] These techniques have been applied also to bilinear quadrilateral elements.

Similar developments have been carried out for triangle elements, using a physical lumping procedure in[11], or the imposition of the Kirchhoff assumption at a set of finite number of points "Discrete - Kirchoff method" in[12] or a free formulation approach as in[13]. The use of drilling degrees of freedom corresponds to the consideration of realistic in-plane rotations and in this way an improved membrane behaviour of the elements is achieved.In reference[14] this idea is applied to the quadrilateral element.

It is possible to obtain shell elements by the general 3-D elements of class C-0. The pioneer work of Ahmad[15] is worthy of mention. By imposing in the 3-D element kinematic constraints among some degrees of freedom, shell elements are obtained. A more recent group of elements based on this idea corresponds to [16]. In this reference a hierarchical finite element family is obtained. Unfortunately the reliability of this type of elements obtained from 3-D elements is low. Selective and/or reduced integration are very efficient techniques but dependent among other variables on the type of computer, problem, loading and geometry of the shell structure. Nevertheless for special group of shells, such as cylindrical shells or axisymmetric shells these elements can be very efficient. An up-to-date report on these type of elements can be seen in[17] An important drawback of these degenerated 3-D elements corresponds to the difficulty of the simultaneous use in an structural analysis of C-1 elements (beams) and C-0 (shell elements)

There are also elements based in mixed energy formulations, i.e., two expansions for the displacement and for the stress fields are used, but it has been shown that most of them can be identified with a reduced-selective finite element formulation in terms of displacements only.

Finally a current active research fields corresponds to obtain an efficient coupling between the elements C-0 and C-1, i.e. plate and slab elements, particularly in relation to the normal rotation (drilling) degree of freedom, of special interest in the coplanar set of elements joined at a common node. In this respect several techniques exist, the first of them being of heuristic type[18]. A recent survey of the state-of-the-art can be seen in[19]. In same cases[20] special displacement fields expansions are used in order to cope reasonably the drilling degrees of freedom. Another possibility corresponds to considering independent fields for the displacements and the drilling degrees of freedom[21]. An interesting extension of this approach is given in the reference[22]

3.- ANALYSIS

Linear (material, small displacements, strains and rotations) and elastic analysis has been the standard in the structural treatment of the shells. However, the importance of the nonlinearities in the structural behaviour of the concrete shells has been observed in many cases[23].

Prestressed and reinforced shell structures have been studied for a long time by the group of the University of California, Berkeley led by Scordelis[24]. The reference[25] is an excellent example of the achievements in this area obtained by this group. The main problems in relation to the material description is about the degree of the refinement to be used in the analysis. Several approaches have been proposed such as the multilayer, or an aggregated (section level) concrete discretization.

Fundamental issues pertaining to nonlinear analysis of shell structures have been carried out by Ramm el alia. They have investigated the aspects related to the efficient techniques for finding eigenvectors and eigenvalues in linear stability problems, to trace the nonlinear shell behaviour (type of the instability, post bucking response etc). Usually the method of inspection of the determinant of the tangent stiffness matrix by the use of the bisection is very efficient[26]. Other very general techniques can follow very closely the nonlinear behaviour of a shell structure but they can not be efficiently applied to find directly the critical instability points[27]. Hangai has shown a very clear technique to classify the different types of instabilities by introducing the concept of generalized matrix inverse[28] . In a future IASS event to take place in Japan in 1993 under the sponsorship of the Seiken Institute of the University of Tokio special emphasis will be placed in the recent progress towards the solution of these problems.

Another class of problems appearing in the analysis of shell structures corresponds to the dynamics of shell behaviour. Response of these structures to wind, earthquake and in some special cases to moving loads can be complicated if nonlinearities have to be included. The situation can be in many cases becomes more difficult if interaction between the shell and another medium has to be analyzed. Tanks and storage structures represent typical examples of these cases. The introduction of the Westergaard formulae, in some instances the slash phenomenum can very relevant[29], added mass concepts, or directly the total coupling in the framework of a single Finite element analysis depend in each case on the importance of the structures and the interaction effects. A recent survey of these problems is given in[30]. Similar problems can appear in the analysis of the soil-shell interaction. In this case, the use of the combination of Finite elements and boundary elements[31] can be more efficient than the standard single Finite element analysis[32], particulary if radiation reflections on the boundaries of the numerical model can take place, as is the case in dynamic problems. As is well known, special boundaries (reflecting boundaries)[33] have to be considered and some of them are not of common use in the engineering practice. Sometimes the use of infinite elements, first introduced in[34] and extended to plate elements in[35] can be applied mainly for static problems, i.e. in order to study a specific area of the shell with a broad consideration of the remaining one.

4.- INPUT DATA

Until relatively recent the input data for the Finite Element analysis was a very tedious task, human time consuming and therefore with an inherent limitation on the total number of degrees of freedom. In this method it is necessary, contrary to the Boundary element method, that all the internal nodes, element connectivity, material properties, loading, boundary geometry and conditions and so on be specified. The difficulties of this man-machine communication have been smoothed due the improvement in the general computer software (friendly input) and in the hardware (tablet, mouse, electronic pencil etc). More recently, the possibility of the combination of graphic software such as AUTOCAD, CADKEY etc with the input data for Finite element analysis is a very common feature for most of the general purpose commercial FE computer programs. In this way, the design of the shell structure, i.e. its geometric definition can directly used for the FE analysis if some indications about the mesh to be used are given or automatically implemented in the software. Therefore, current input data for FE analysis can be divided into the three groups:

-Graphic input
-Automatic mesh generation
-Adaptative mesh

The first group is directly dependent on the development of the computer hardware, and hence it will not be here discussed.

The second group has started by special techniques (languages, macro-instructions etc) created to describe FE meshes. More sophisticated procedures have been developed in order to diminish the human user intervention in the mesh design. Reference[36] represents a very complete description of the situation of the current state of the field.

Techniques mainly addressed to the solution of 2-D problems such as isoparametric, lagragian, conforming transformations are well established. Extension of automatic FE generators to shell and 3-D structures represents a formidable task. Several special treatments have been developed in this field, and they can broadly be classified as follows:

(a)-Structured meshing.
(b)-Non-structured or unstructured meshing.
Inside this last group it is possible to distinguish two main subgroups: (b1) Macro non-structured meshes or multiblock (they consist of groups of structured meshes). (b2) Micro non-structured meshes.

The techniques to obtain a mesh belonging to the group (a) are very complicated to program if shell and 3-D structures of arbitrary geometry have to be automatically meshed.

However, the techniques producing meshes of the group (b) demand a more user interaction, but more efficient meshes can be obtained than the ones from the previous procedures. Several micro-structured methods have been published recently, such as the ones called the triangularization of the Delaunay[37], advancing front[38], recursive subdivision[39], octree and modified octree[40] among others.

A further advance in the input of the FEM is to try to obtain a mesh that within a given number of degrees of freedom, can produce "best" results from the analysis, according to a given objective function (optimal mesh). A recent publication[41] presents some examples of this approach. Also, one of the weak points of the standard practice of FEM is the lack of procedures to evaluate the goodness and even the validity of the results obtained from the analysis. Engineering intuition, comparison with alternative analysis and experience have been for a long time the typical approach to this situation. For some years there are several techniques to evaluate the performance of a FE analysis in terms of the accuracy of the results The "a posteriori error analysis", started by Babuska et alia represents a practical answer of the subject on the FE quality control.

In the current state of the use of the FE applications, the "a posteriori error estimators" can be used to assess the validity of an analysis, if some allowable maximum error is assumed. An efficient or optimal mesh is defined in this context if the estimated a posteriori error is nearly constant trough out the whole structure under consideration. Therefore, an iterative procedure (called adaptative meshing) can be performed in order to obtain the best mesh. After a FE analysis the errors are estimated and the parts of the structures where the estimated error are larger than the assumed limits are remeshed. In this way, this step by step procedure stops when the whole structure presents an estimated error nearly constant and below the allowable one.

There are several procedures to produce a new better mesh and they are related with the type of convergence in the FEM: h, p and h-p convergences. If the dimension of the element is diminished normally by the introduction of a new node the convergence is termed h-convergence. By the contrary, if the order of the polynomial expansion of the element displacement field is increased, the convergence is denominated p-convergence. A simultaneous use of both types of mesh refinements is the so called h-p convergence. Usually the p-convergence is more rapid than the h-convergence.

Normally, the adaptative mesh procedure just described is carried out in a few steps due to use of some theoretical estimation of the size or the polynomial degree to be use in the region of the structure to be remeshed. In this way the level of remeshing is obtained and therefore the number of extra nodes or order of the polynomial expansion is directly computed in a single step in order to reach an allowable error. It is important to realise the fact that these theoretical estimations of the mesh (sizes or/and polynomial degrees) can be very dependent on the level of the singularity of the problem. That is, it can be very sensitive in some cases of the shell analysis due to the existence of point supports, stress concentration, isolated loads etc.

In the state-of-the art of the "a posteriori error analysis" there exist three main class of methods:

(1).-The error estimators of the residual type. It was the first proposed in the literature[42] and it is now days well established.

(2).-Error estimators of postprocess type. It is used a projection or recuperated solution from the results of the computer analysis of the structure. This projected or smoothed solution represents a higher order solution than the one obtained from the usual FE analysis and it can be used as the exact solution in order to estimate the errors. These Z-Z estimators have been described in[43] and several proposed smoothing techniques like averaging the stress field or the interpolation by a minimum quadratic error have been discussed and proved to be efficient.A specific application to shell structures is given in the publication[44], where the Z-Z techniques are applied to Kirchhoff and Middlin (C-1) shells.

(3).-Finally, the estimators based in a priori interpolations produce very poor results but they are simple to implement into the computer and they are also very useful as error indicators.

Recently, the idea of Z-Z estimators is now under progress, i.e. to develop techniques to recuperate locally FE solutions with a very high degree of convergence (superconvergence) from the FE standard analysis. Several authors have studied this superconvergence phenomenon and the reference[45] represents an excellent review of the current state-of-the-art.

5.- OUTPUT RESULTS

Typical FE results for shell structures are related to the displacement and stress fields. Voluminous numerical output, which entails painstaking reduction of data is not longer given in commercial computer FE programs. Reduction of the results in ordered form and maxima-minima description, contour lines, principal stresses are common features in the nowadays engineering practice. Moreover, graphical output using plotted results or coloured states of displacements and stresses are normally included in a general FE computer program. Despite of these improvements, shell structural analysis demands more postprocessing, particularly with respect to the difficult issue of the reinforcement design needed to resist the stresses obtained from a standard linear FE analysis. Gupta[46] and Medwadowski[47] have treated these problems, for in-plane forces. The extension of these results to the general stress-resultants can perhaps be handled following the same lines.

6.- COMPUTER PROGRAMS

The path already done since the first FE analysis in the fifties until the present situation has been considerable and the computational efficiency reached in all phases of the FE programming (input data, memory allocation, stiffness computation and assembly, system of equations solvers, stress recovery and output presentation) has been extraordinary. In many cases, this software progress has followed the improvement in the hardware, and now it is possible to run a quite sophisticated FE program in an easily available PC computer. This situation will be changed even more, when parallel computation approach gains popularity and spread out through the academic and professional community.

The impact of the parallel processing into the FE analysis will be extraordinary, in all already described phases of the computer programming. An recent revision of this impact is given in the reference[48] Parallel processing introduces its own philosophy in different areas of the programming such as the data structure, memory requirements, communication among the different processors and the demand for a specific parallel arithmetic[49]. Each processor in a FE analysis may be represents a stiffness matrix prior its assembly or simply a nodal point without assembling. These aspects are discussed in detail in[50]

7.- EXPERT SYSTEMS

In the future, and already at the present time in some cases, the design and analysis of shell structures, can be handled perhaps by an Expert System. Some trials in this direction are now under way with different success. An excellent review of the current situation is described in the book[51] where from typical computer aspects like suitable programming languages to be used up to preliminary strategies to consider during the design of an Expert System are discussed.

A more modest, but important, task to be handled in the computer is related to the shape finding methods of shells. A large amount of the technical literature is devoted to this important aspect of the design. Medwadowski in[52] reports the evolution of shell structures, from the historical trial and error approach up to the modern era with new material, industrial processes and analytical and computer analysis have produced dramatic changes on the shell design. But despite these changes shell design is still restricted, except in some very outstanding examples of designers like Gaudí, Torroja or Isler, to very regular forms. Optimal design techniques may be the right answer to introduce a variety in the shell forms. The obvious technique is to generate the shell shape by computer simulation of the Fialho[53] experiments. For a given arbitrary plan a flat membrane is subject to the acting loads and in this way the membrane is deformed and the desired form is obtained. The simulation technique can be the large deflection membrane theory, using FE analysis and the optimal form is obtained by reversal of the deflected form, i.e. changed from the pure tension state to the pure compression state for the concrete shell. Computer details can be see in[54] The main problem with this procedure is to find a compromise, when different load cases are dominant, but fortunately that is not the usual situation in many shell structures (roof shells).

The form finding of shells leads naturally to mathematical programming schemes. The objective function may be the total weight, but in concrete shell structures this objective is not very significant. Alternative objective functions can be the formwork area, total cost or full compressive stress state below some design limit. The different load cases can in this method be treated by individual weighting of the individual cases in the objective function[55].

Finally, in the problem of shape finding of shell structures multicriterion objective functions can be used. The reference[56] shows some aspects of these techniques.

8.- FINAL REMARKS

In this paper a review of the applications of computer and numerical methods to the shell design has been given. Some of these applications seems to be very challenge to our capability as shell designers, but we should remember that historically every step towards a better human performance and interaction with our world has meant more interdependence on others. Therefore we must be aware of the restrictions and limitations of our work and also place our top priority in thinking through our objectives and motivations for choosing to act in a particular direction during the design process.

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